The harvesting effect of Pacific Yew for cancer treatment on the ecosystem: Mathematical modeling approach

Lucero Rodriguez Rodriguez, Yun Kang

Arizona State University

Introduction

- The Pacific Yew is a native North American tree. It is know for its slow growth and low density of trees per hectare[1].
- Breast cancer remains the most diagnosed and most fatal form of cancer among women in the world[2].
- The chemotherapy drug Paclitaxel (Taxol), derived from the needles (leaves), bark, and seeds of the Pacific Yew is used to treat breast, ovarian, and lung cancer[3,4].
- Yew trees have been unsustainably harvested to meet the demand of Taxol, and as a result, many *Taxus* trees are now endangered[5].
- We propose an ecological mathematical model that considers the populations of Pacific Yew trees, breast cancer patients, and breast cancer patients who have recovered after Taxol treatment.

Research Questions

We explore the dynamics between cancer patients and the Pacific Yew population, and focus on how this population affects the Pacific Yew population due to harvesting, specifically:

A. What is the ecological impact of tree harvesting on public health issues?

B. How does overharvesting the Pacific Yew affect cancer patients?

C. How does successful treatment impact the Pacific Yew?

Model

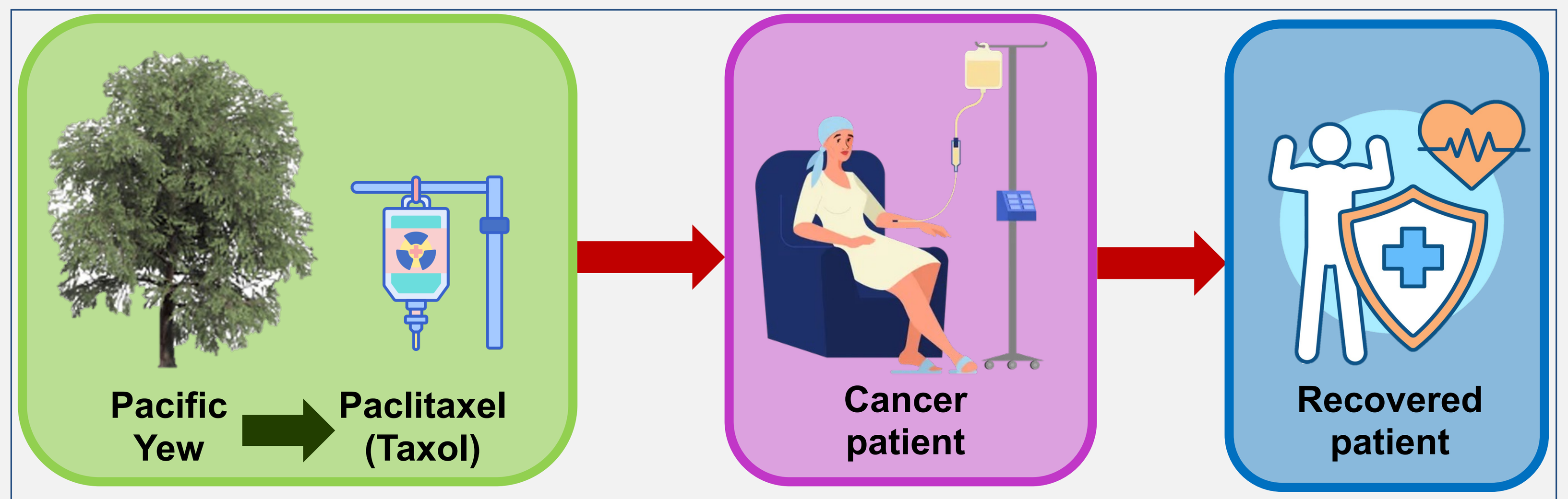


Figure 1. Schematic relationship between Pacific Yew tree, cancer patients and recovered patients.

$$\frac{dP}{dt} = r_P P\left(1-\frac{P}{K_P}\right) - \frac{\alpha C P}{1+\beta P}$$

$$\frac{dC}{dt} = r_C C\left(1-\frac{C}{K_C}\right) - \frac{q\alpha C P}{1+\beta P}$$

$$\frac{dR}{dt} = \frac{q\alpha C P}{1+\beta P} - \mu_R R$$

Parameter	Description
r_P	Yew's max growth rate
r_C	cancer patients max growth rate
K_P	Yew's carrying capacity
K_C	cancer patients carrying capacity
α	Yew's removal rate
β	Yew pop. level at which the cancer patients removal capability begins to saturate
q	Prob treatment is successful
μ_R	Per capita death rate

Model Dynamics

Equilibria & Stability

Equilibrium	Existence	Stability
$\mathbf{E_{0,0,0}}$	Always	Always unstable.
$\mathbf{E_{P^*,0,0}} = (K_P, 0, 0)$	Always	LAS when $\alpha > \frac{r_C(1+\beta K_P)}{q K_P}$, otherwise it is unstable.
$\mathbf{E_{0,C^*,0}} = (0, K_C, 0)$	Always	LAS when $\alpha > \frac{r_P}{K_C}$, otherwise it is unstable.

Global Dynamics

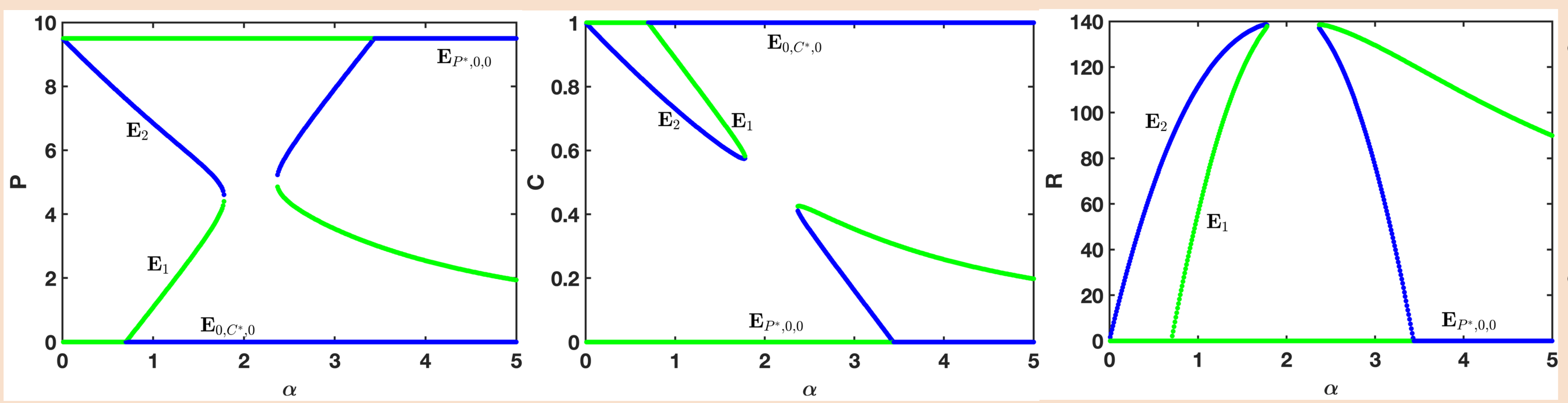
1. Persistence of Tree Population
2. Persistence of Cancer Population
3. Bistability between:

- *Disease-free equilibrium and tree-free equilibrium.*
- *Tree-free equilibrium and interior equilibrium.*

Bifurcation Diagrams

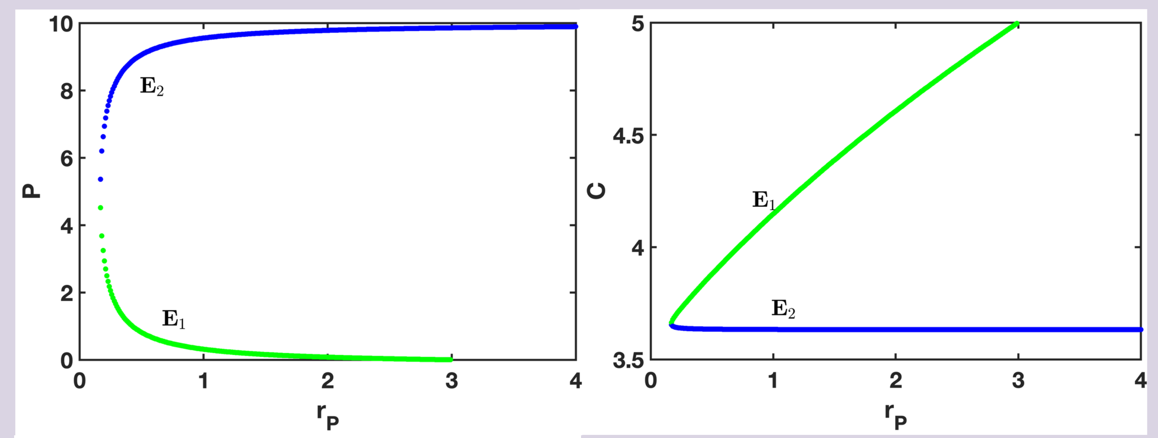
Theorem 1. If $q < \frac{r_C K_C}{r_P}\left(\frac{1}{K_P} - \beta\right)$, Model (1) undergoes a backward (subcritical) bifurcation at $\alpha = \frac{r_P}{K_C}$ around the tree-free equilibrium, otherwise, it undergoes a forward bifurcation.

The diagrams show the interior equilibria of the system, where blue represents stable equilibria and green indicates saddle points.



- Small perturbations in the initial population sizes of the Yew tree and the cancer patients can lead to tree extinction.
- High initial population is needed for the Yew tree population to persist.

Figure 2. One parameter bifurcation for our model as a function of Yew's removal rate.



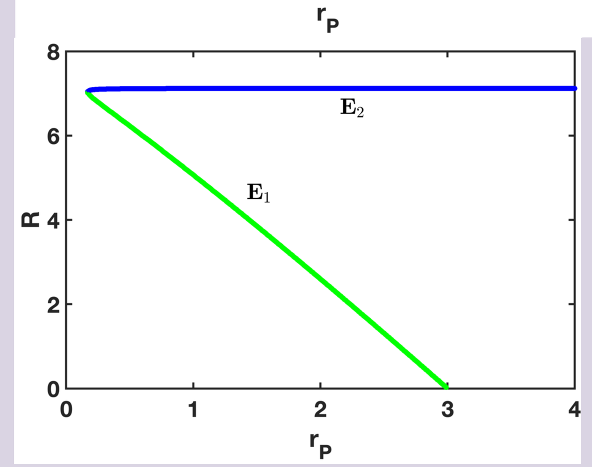


Figure 3. One parameter bifurcation for our model as a function of the Yew's max growth rate.

- For higher harvesting rate we require a smaller cancer patient population size for coexistence.
- If treatment efficacy is less than the impact of cancer patient growth to the tree population and the impact of removal capabilities on the tree population there is a <u>backward bifurcation</u> when the harvesting rate is equal to the ratio of the tree's growth rate and the cancer patient population.
- Increase in the Yew's growth rate can stabilize and preserve the tree population and continue to be harvested to produce chemotherapy.
- Increasing seed dispersion and germination of the tree will increase the growth rate and allow the conservation of the species.

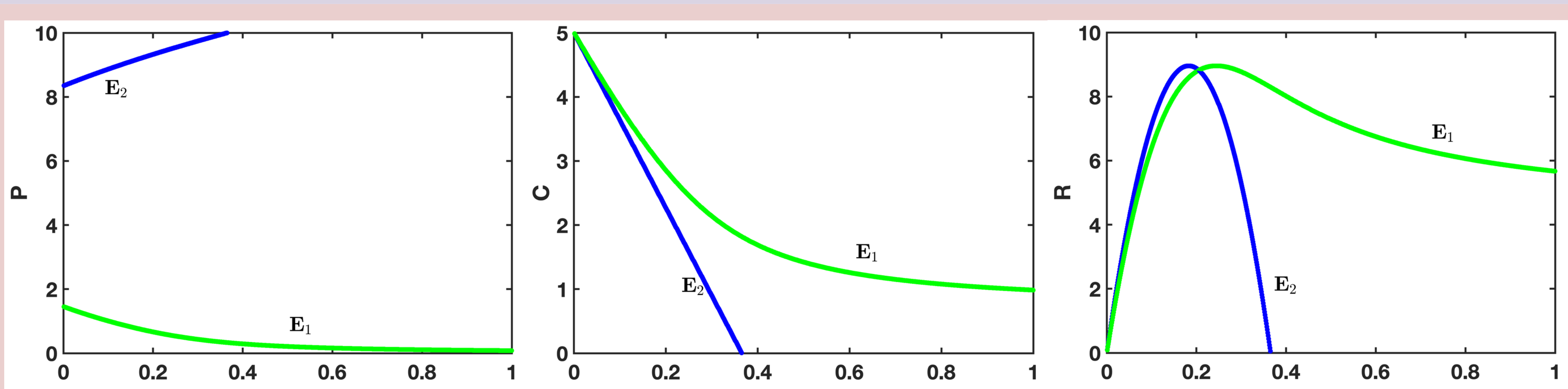


Figure 4. One parameter bifurcation for our model as a function of the probability treatment is successful.

- Only the Yew tree population will persist as treatment efficacy reaches 36%.
- In this scenario, the optimal treatment efficacy is of 18%, where the number of recovered patients is maximized.

Conclusions

<u>Analytical</u> results

- In the absence of a stable interior equilibrium, the system will converge to either the disease-free or tree-free equilibrium, with one becoming globally stable.
- Our model has up to two interior equilibria, allowing the coexistence of Pacific Yew trees and cancer patients.
- Tree population can go extinct due to overharvesting, high cancer patient density, and a low tree population.

<u>Numerical</u> results

- Overharvesting drives the Pacific Yew population to extinction.
- Maximizing treatment efficacy can benefit both the Pacific Yew tree population and the coexistence of cancer patients.
- Measures for seed dispersion and germination can help conserve the species while continuing its use as a resource

Future Scope

The model can be modified to account for:

- Susceptible population with risk of developing breast cancer.
- Recurrent breast cancer
- Regulations for harvesting of the Yew tree.

Acknowledgements

This research was sponsored by the Research Assistantship awarded by the School of Human Evolution and Social Change from Arizona State University.

References

1. R. Burns, B. Honkala, U. S. F. Service, Silvics of North America: Conifers, Agriculture handbook, U.S. Department of Agriculture, Forest Service, 1990.
2. H. Sung, J. Ferlay, R. L. Siegel, M. Laversanne, I. Soerjomataram, A. Jemal, F. Bray, Global cancer statistics 2020: Globocan estimates of incidence and mortality worldwide for 36 cancers in 185 countries, CA: A Cancer Journal for Clinicians 71 (2021) 209–249.
3. G. A. Strobel, A. Stierle, F. J. van Kuijk, Factors influencing the in vitro production of radiolabeled taxol by pacific yew, taxus brevifolia, Plant Science 84 (1992) 65 – 74.
4. S. Mayor, Tree that provides paclitaxel is put on list of endangered species, 2011.
5. S. Mayor, Tree that provides paclitaxel is put on list of endangered species, BMJ (Clinical research ed.) 343 (2011) d7411