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Spherical Harmonic Applications in Cosmic Microwave Background

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Cosmic Microwave Physics

In the simplest of terms, the CMB informs us about how our universe could have formed following expansion and cooling. During expansion, photons in the CMB stretch with the universe as they are unbounded from electromagnetic forces. Moving forwards causes the universe to fill with a plasma consisting of free electrons, nuclei, and photons. Particles in the plasma interact with electromagnetic forces and Thomson scatter to create photon-baryon fluids. Fluids move similarly to gasses and are subjected to compressive and rarefactive oscillations (sound waves) as they move into gravitational potential wells (Wayne Hu).

While the topic of universe development can be continued until the next universe development, the scattering of photons is what we are looking at when analyzing CMB temperatures. The fluctuation in photon-baryon oscillations creates the temperature information depending on how fast or slow the oscillations occur at. Different gravitational potential wells have different oscillatory information based on how deep and high the space is.

Spherical Harmonics

The DMR instrument is designed to measure temperature fluctuations between different parts of the sky. For this to be done, the instrument takes in measurements based on angular degrees to create a temperature distribution using spherical harmonics.

Spherical Harmonic Equation:

$$Y_\ell^m(\theta,\phi) = (-1)^m \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\,P_\ell^m(\cos\theta)e^{im\phi}$$

Expansion of Function with Spherical Harmonics:

$$f(\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=\ell}^{\ell} a_{\ell m}Y_\ell^m(\theta,\phi)$$

Coefficient Equation:

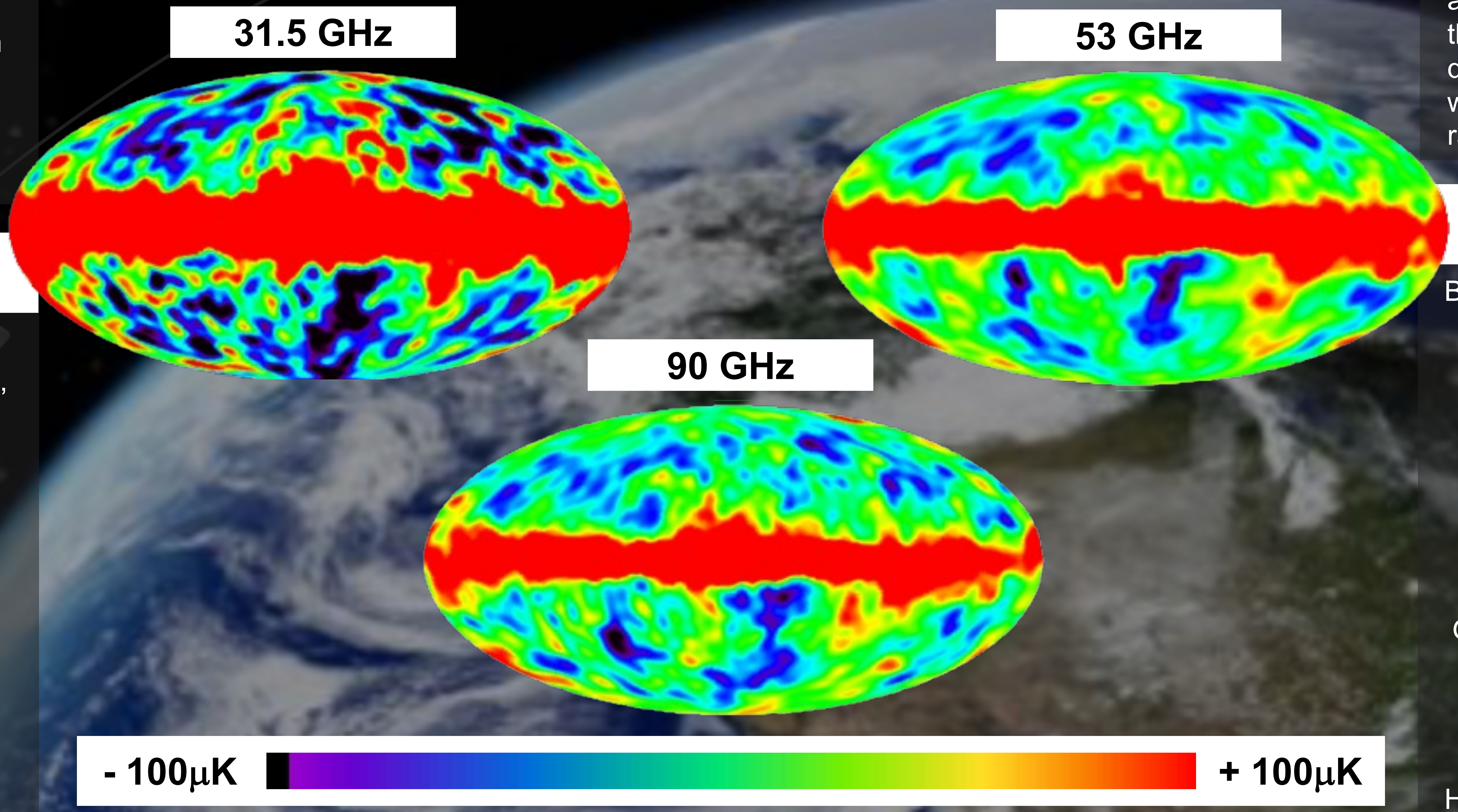
$$a_{\ell m} = \int_{-1}^{1}\int_{0}^{2\pi}\left[Y_\ell^m(\theta,\phi)\right]^* f(\theta,\phi)d\phi d(\cos\theta)$$

It should be noted that for our application, DMR readings only depend on angular values. Directional dependence (*m*) vanishes.

Introduction

The Cosmic Background Explorer (COBE) was sent into space on November 18, 1989. The satellite was equipped with three instruments: Diffuse Infrared Background Experiment (DIRBE), Far Infrared Absolute Spectrophotometer (FIRAS), and the Differential Microwave Radiometer (DMR). Each instrument was uniquely created to measure and compare features of the Cosmic Microwave Background (CMB) with a precise blackbody radiation. As one of the first satellites created to measure the CMB, COBE operated using large-scale anisotropy parameters to produce distributions based on large angular scales. For this project we will numerically analyze data from the Differential Microwave Radiometer (DMR) using spherical harmonics as our anisotropy parameters.

DMR Temperature Sky Maps



Temperature Map Analysis

The following maps show DMR data taken by an angular degree of 10 and include the frequencies of 31.5 GHz, 53 GHz, and 90 GHz. Each map is contrasted and subtracted by a dipole to ensure fluctuations are present. Without contrast or dipole subtraction, the maps would appear uniform throughout the entire map. The red stripe across the middle shows the Milky Way Galaxy. This means that the temperatures surrounding the top and bottom illustrate the CMB from a measured angular horizon (LAMBDA).

Conclusion

The CMB was first theorized by George Gamow in 1948 when he argued that the universe was a product of thermonuclear reactions. Then in 1964, Robert Wilson and Arno Penzias experimentally discovered the CMB and thought it to be uniform in all directions. Currently, CMB satellites can measure and produce robust power distributions using spherical harmonics. While COBE was limited during its time, it set forth the foundations for understanding what the CMB has to offer in the cosmological field of study. The theorization and experimentation, along with the ability to understand particle structures, allowed for the knowledge that we now have. NASA is still on the quest to understand what has yet to be discovered when looking at the comic microwave background radiation.

References

Bennett, C. L., et al. “Four-Year COBE DMR Cosmic Microwave Background Observations: Maps and Basic Results.” *The Astrophysical Journal*, vol. 464, no. 1, 1996, pp. L1–L4, https://doi.org/10.1086/310075.

Wright, E. L., et al. “Angular Power Spectrum of the Cosmic Microwave Background Anisotropy Seen by the COBE DMR.” The Astrophysical Journal, vol. 464, no. 1, 1996, pp. L21–L24, https://doi.org/10.1086/310073.

Górski, K. M., et al. “Power Spectrum of Primordial Inhomogeneity Determined from the FOUR-Year COBE DMR Sky Maps.” The Astrophysical Journal, vol. 464, no. 1, 1996, pp. L11–L15, https://doi.org/10.1086/310077.

Hu, W. (n.d.). *CMB Tutorials*. Wayne Hu's Tutorials. http://background.uchicago.edu/index.html

Greason, M. R. (Ed.). (n.d.). *Cosmic Background Explorer*. NASA. https://lambda.gsfc.nasa.gov/product/cobe/

Knox, L. (2022, June 21). *Workbook*. Physics LibreTexts.https://phys.libretexts.org/Courses/University_of_California_Davis/UCD%3A_Physics_156_-_A_Cosmology_Workbook/Workbook