Comparing Student Strategies in a Game-Based and Pen-and-Paper Task for Linear Algebra

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Introduction and Objectives

- Overarching Motivation: When does it make sense to use *a game* as opposed to some other engaging interactive task?
- In Linear Algebra – two related tasks, the Magic Carpet Ride (MCR) task (Wawro et al., 2012; Wawro et al., 2013) and *Vector Unknown* (VU) (Mauntel et al., 2021) offer a good case study to explore:
  - MCR task created first as part of Inquiry Oriented Linear Algebra (IOLA) curriculum (Wawro et al., 2013)
  - VU is a game based on the MCR task (Mauntel et al., 2021)
  - Thus: we have a game-based and non-game based task that are centered on similar learning objectives

The Tasks

Vector Unknown

Players use vectors to control the bunny, placing them into the vector equation at top and then making a linear combination.

Magic Carpet Ride

Vectors represent the movement of a hoverboard and magic carpet. Students figure out to travel on each to Old Man Gauss.

We denote the restriction on the *hover board's* movement by the vector $\begin{bmatrix}3\\1\end{bmatrix}$.
By this we mean that if the hover board traveled "forward" for one hour, it would move along a "diagonal" path that would result in a displacement of 3 miles East and 1 mile North of its starting location.

We denote the restriction on the *magic carpet's* movement by the vector $\begin{bmatrix}1\\2\end{bmatrix}$.
By this we mean that if the magic carpet traveled "forward" for one hour, it would move along a "diagonal" path that would result in a displacement of 1 mile East and 2 miles North of its starting location.

SCENARIO ONE: THE MAIDEN VOYAGE

Your Uncle Cramer suggests that your first adventure should be to go visit the wise man, Old Man Gauss. Uncle Cramer tells you that Old Man Gauss lives in a cabin that is 107 miles East and 64 miles North of your home.

TASK:
Investigate whether or not you can use the hover board and the magic carpet to get to Gauss's cabin.
If so, how? If it is not possible to get to the cabin with these modes of transportation, why is that the case?

Research Questions

- We could explore our overarching motivation by comparing many different aspects – e.g., affect, engagement, outcomes
- We wanted to look at the mathematical engagement of students , which we operationalized as strategies, with the following RQs:
- RQ 1: What strategies do students deploy in the VU game and the Magic Carpet Ride task, and how can they be categorized?
- RQ 2: What patterns are apparent in the use of strategies across these tasks?

Strategies Used in the Tasks

Strategy	Definition
Experimenting with Scalars and Vectors	The student tries using different vectors and scalars to explore potential solutions or pathways to solutions.
Sketching a Graph	The student creates a graph, not necessarily to scale, which includes at least some of the following: labeled axes, movement vectors, the goal position, and/or lines based on the vectors or goal position.
Linearly Independent Vector Selection	The student selects two vectors to work with by eliminating linearly dependent vectors (e.g. scalar multiples).
Quadrant-Based Reasoning	The student uses the quadrant of movement vectors and/or the slopes of lines defined by the movement vectors and compares them to the goal position to inform their work.
Focus on One Coordinate	The student reduces the aim of their work to reaching one coordinate and gets as close as possible to that coordinate, then adjusts to match the other coordinate.
Focus on One Vector	The student uses one vector to get as close as possible to the goal position before introducing a second vector and using both to reach the goal.
System of Equations	The student uses two vectors and the goal position to model the problem as a System of Equations with two equations and two unknowns, and then solves this System of Equations using any valid method.

Participant	Rikaine		Pat		Terry		Auren		Chris	
Task	VU	MCR	VU	MCR	MCR	VU	MCR	VU	MCR	VU
Sketching a Graph		G		G	G				G	
Experimentation with Scalars and Vectors	G		G		N	G				G
Quadrant-Based Reasoning	G, N	N			A, N	G, N		G, N		
Focus on One Coordinate	N		G, N	N			N	N		N
Focus on One Vector	G, N	N	G, N	N		N	N	G, N		
System of Equations				A			A		A	A
Linearly Independent Vector Selection	N		N					N		N

(Non-)Repeated Strategies

- Many strategies got used by participants in both MCR and VU, such as how Pat used Focus on One Coordinate:

Vector Unknown

Pat: Alright, so um, x is negative 10, y is four. So, in order to get there, moving 10 back first…Let's just start off with this [<-2, 3>], so. [Increases scalar in front of <-2, 3> to 5] So that's negative 10...

Magic Carpet Ride

Pat: So how do I get to (107, 64). Let's see about getting to 107 first…35 hours in this direction. We'll get to 105, 35. Of course I wouldn't be able to get to...

$35\langle 3,1\rangle = \langle 105,35\rangle$

Pat: I wonder if it's better to get to 64 first. So if it's 32 hours by magic carpet, that's the point (32, 64). Which would arrive me at the y value of his house.

$32\langle 1,2\rangle = \langle 32,64\rangle$

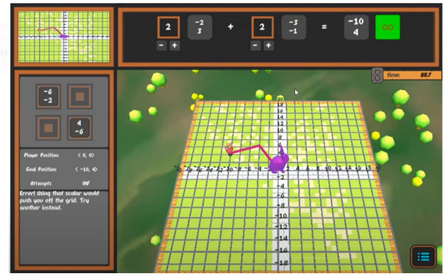
- Even as participants would try some of the same strategies with both tasks, there were some they would deploy in only one – for example, Pat would ultimately set up and solve a System of Equations in the MCR task, whereas they stuck with Focusing on One Coordinate (& Focusing on One Vector) in VU:

$x\langle 3,1\rangle + y\langle 1,2\rangle = \langle 107,64\rangle$

$3x + 1y = 107$
$1x + 2y = 64$

Magic Carpet Ride

Pat: I suppose I could set up a simple system of equations...So y = 107 - 3x. Plug that into the bottom one and I get x + 2(107 - 3x) = 64. x plus two 107 so 214 yeah 214 - 6x = 64. -5x = 64 – 214. [inaudible mumbling, writing on their sheet of paper]…Yeah so x = 30, if I go ahead and plug that back in. 3*30 + y = 107. So 90 + y = 107. So y = 17.



Vector Unknown

Pat: So that's negative 10. Now reversing that to get to four currently at sixteen no that's fifteen...Let's see...Hopefully this [selects <-3, -1> vector] is what I want. [Alternates increasing the scalar on <-3, -1> and decreasing the scalar on <-2, 3> to reach the goal] Yup, there we go.

References and Acknowledgements

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